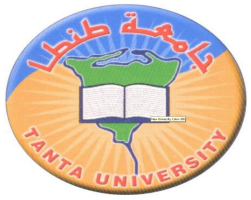

EEC 4230 - Mobile Communication Systems

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Lecture 3: Trunking, Grade of Service (GoS), Improving Coverage and Capacity in Cellular Systems

Outline

1. Trunking and Grade of Service (GoS)
2. Improving Coverage and Capacity in Cellular Systems

Trunking and Grade of Service (GoS)

Trunking:

- It refers to sharing few channels among many users.
- Let U be the # of users, and C be the # of channels.
- Each user requires a channel infrequently (we call <30% of the time),
- So we exploits statistics of call-request distribution (all users will not request calls at the same time).
- So a dedicated channel for each user is not required.
- Also, the request for a channel happens at random times,
- So each user is allocated a channel on a per call basis,
- Upon termination of the call, previously occupied channel is immediately returned back.
- So for any $C < U$, it is possible that there will be more requests than channels.
- When user requests service and all of channels are already in use, the user is blocked.

Grade of Service (GoS) (or Blocking Probability):

- Defined as the prob an offered call will be blocked (and not served, or carried by system).
- It is a measure of the user's ability to access a trunked system during the busiest hour.

Trunking and Grade of Service (GoS)

The trunking theory was developed by Erlang to design trunked radio system of specified capacity & specified GoS.

Deinitions:

- **Erlang**: A "unit" of measure of usage or traffic intensity. One Erlang is the traffic intensity carried by one channel that is occupied all of the time. 0.1 Erlang is the same channel occupied only 10% of the time.
- **Average holding time (H)**: Average call duration of a typical call.
- **Call rate (λ)**: Avg # of call requests(arrival) per unit time (per user). Typically taken to be at the busiest time of day.
- **Traffic intensity offered by a user**: $A_u = \lambda H$.
- **Total offered traffic intensity for a system containing U users**: $A = UA_u$
- **Traffic intensity per ch in C trunked system**: $A_c = \frac{A}{C}$ and C is the # of chs
- **Setup time**: Time required to allocate a trunked channel to a requesting user.
- **Blocked (lost) call**: which cannot be completed due to congestion
- **GoS**: blocking Prob of a call (for Erlang B), or prob of a call being delayed beyond a certain amount of time (for Erlang C).

Trunking and Grade of Service (GoS)

Example 1:

If a user makes on average, two calls per hour, and that call lasts an average of 3 minutes. Assume that there are 1000 users and 200 channels and calculate:

1. The traffic intensity offered by a user A_u,
2. The total offered traffic intensity A,
3. the total offered traffic intensity per channel A_c,

Solution:

1. $A_u = \frac{2}{60min} 3min = 0.1$ Erlang
2. $A = UA_u = 1000(0.1) = 100$ Erlang
3. $A_c = \frac{A}{C} = \frac{100}{200} = 0.5$ Erlang

Note that A_c is a measure of the efficiency of the utilization of the channels.

Trunking and Grade of Service (GoS)

How should we design our system? (How should we set U, A_u, C to satisfy our customers?)

Obviously, Ac should be less than 1 ($A < C$); or we'll never satisfy our call demand. What do we do when a call is offered (requested) but all channels are full?

(1) Blocked Calls Cleared systems (or Erlang B): Igonre it!
(2) Blocked Calls Delayed systems (or Erlang C): Postpone it!.

(1) Blocked calls cleared model (Erlang B):

- No queuing for call requests, the user is given immediate access to a channel if one is available, or it is blocked if none is available, and the user is free to try again later.
- Consider a system with C trunked channels
- Assume calls requests n are a Poisson distribution. That is times between calls τ_n are exponentially distributed, and memoryless. Call durations τ are also exponentially distributed.
- Poisson process is generally used to model the aggregate traffic of a large number of similar and independent users accessing a system. So, let $A(t)$ = number of call requests arriving in the system at time t, then $P\{A(t+\tau) - A(t) = n\} = e^{-\lambda\tau}\frac{(\lambda\tau)^n}{n!}$

Trunking and Grade of Service (GoS)

(1) Blocked calls cleared model (Erlang B):

- $\lambda\tau$: is the total average number of arrivals within time interval τ (for all users)
- Inter arrival time is exponentially distributed with pdf $p(\tau_n) = \lambda e^{-\lambda\tau_n}$
- The avg inter-arrival time (period during which channels are idle) is given by $\frac{1}{\lambda}$
- For a finite number U of users each requesting A_u traffic, we set the total offered traffic as a constant A, and then let $U \longrightarrow \infty$ and $Au \longrightarrow 0$ in a way that preserves $UA_u = A$. This is the "infinite # of users" assumption that simplifies things considerably.
- These assumptions, along with setup of the system, leads to the Erlang B formula.

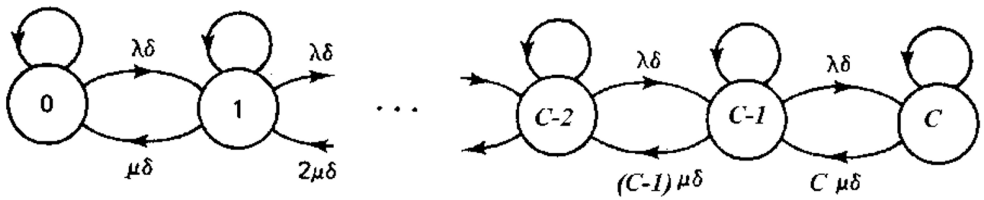
$$GoS = P(blocking) = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C} \frac{A^k}{k!}}$$

Since C is very high, it's typically easier to use Figure on slide 10. By setting the desired GoS, we can derive what # of channels we need; or maximum # of users we can support (and set the # of minutes on our calling plans accordingly).

Trunking and Grade of Service (GoS)

(1) Blocked calls cleared model (Erlang B): Proof [see appendix A in textbook]

- Let average duration of call (average service time) be $H = \frac{1}{\mu}$ where μ is the service rate.
- The prob of blocking is the steady-state prob that all channels are busy, in which case an arriving call is blocked.
- The system can be modeled as a discrete-time Markov chain with state transition diagram shown below:



where δ denotes discrete-time index, and $\lambda\delta$ is the average number of arrivals in the time interval δ and μ denotes the service rate, i.e. $\mu = \frac{1}{H}$

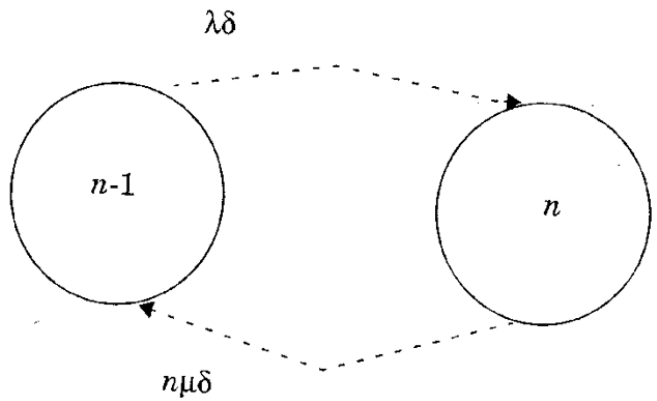
Trunking and Grade of Service (GoS)

(1) Blocked calls cleared model (Erlang B): Proof [see appendix A in textbook]

- State 0: all channels are idle and ready for use
- State 1: 1 channel in use, $C-1$ channels idle
- :
- State C: C channels are in use, 0 channel idle (blocking occur for new arrivals).

Over a long period of time, the system reaches steady state and at steady state, we can write:

$$P_1 = P_0\frac{\lambda}{\mu}, P_2 = P_1\frac{\lambda}{2\mu}, ..., P_C = P_{C-1}\frac{\lambda}{C\mu}$$



Trunking and Grade of Service (GoS)

(1) Blocked calls cleared model (Erlang B): Proof [see appendix A in textbook]

- In General: $P_n = P_0(\frac{\lambda}{\mu})^n \frac{1}{n!}, \forall n = 1, 2, ..., C$ (Eq. A.23)
- Also, $\sum_{n=0}^{C} P_n = 1$
- Thus $P_0 + P_0(\frac{\lambda}{\mu})\frac{1}{1!} + P_0(\frac{\lambda}{\mu})^2\frac{1}{2!}... + P_0(\frac{\lambda}{\mu})^C\frac{1}{C!} = 1$
- Therefore, $P_0 = [\sum_{n=0}^{C}(\frac{\lambda}{\mu})^n\frac{1}{n!}]^{-1}$ (Eq. A.25)

The prob that an arrival will find all C channels busy (and will be blocked):
$P_C = P_0(\frac{\lambda}{\mu})^C\frac{1}{C!}$ (Eq. A.26)

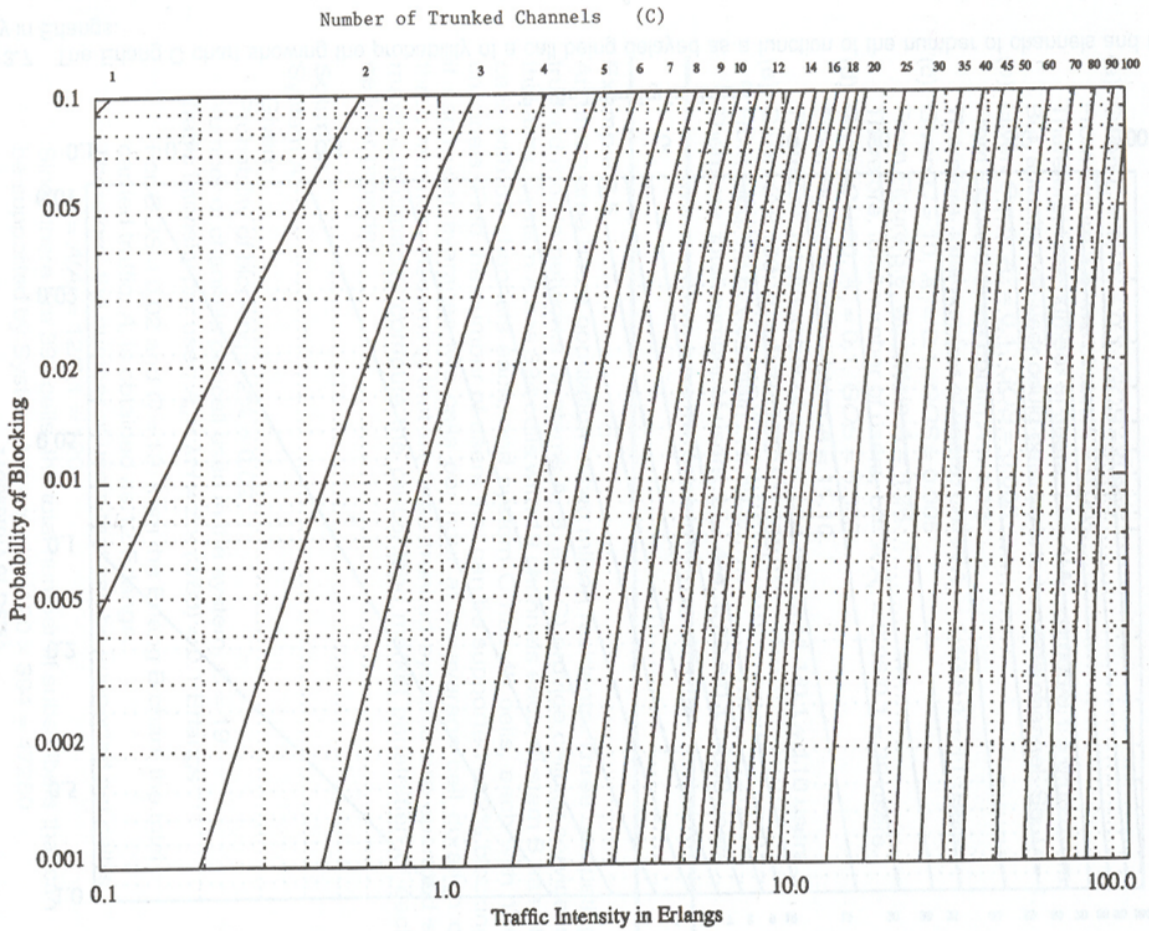
Substituting Eq. A.25 into A. 26 yields: $P_C = \frac{(\frac{\lambda}{\mu})^C\frac{1}{C!}}{\sum_{k=0}^{C}(\frac{\lambda}{\mu})^k\frac{1}{k!}}$

Note that $H = \frac{1}{\mu}$, and $A = \lambda H = \frac{\lambda}{\mu}$

Therefore, $GoS = P(blocking) = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C}\frac{A^k}{k!}} \Rightarrow$ Erlang B Formula.

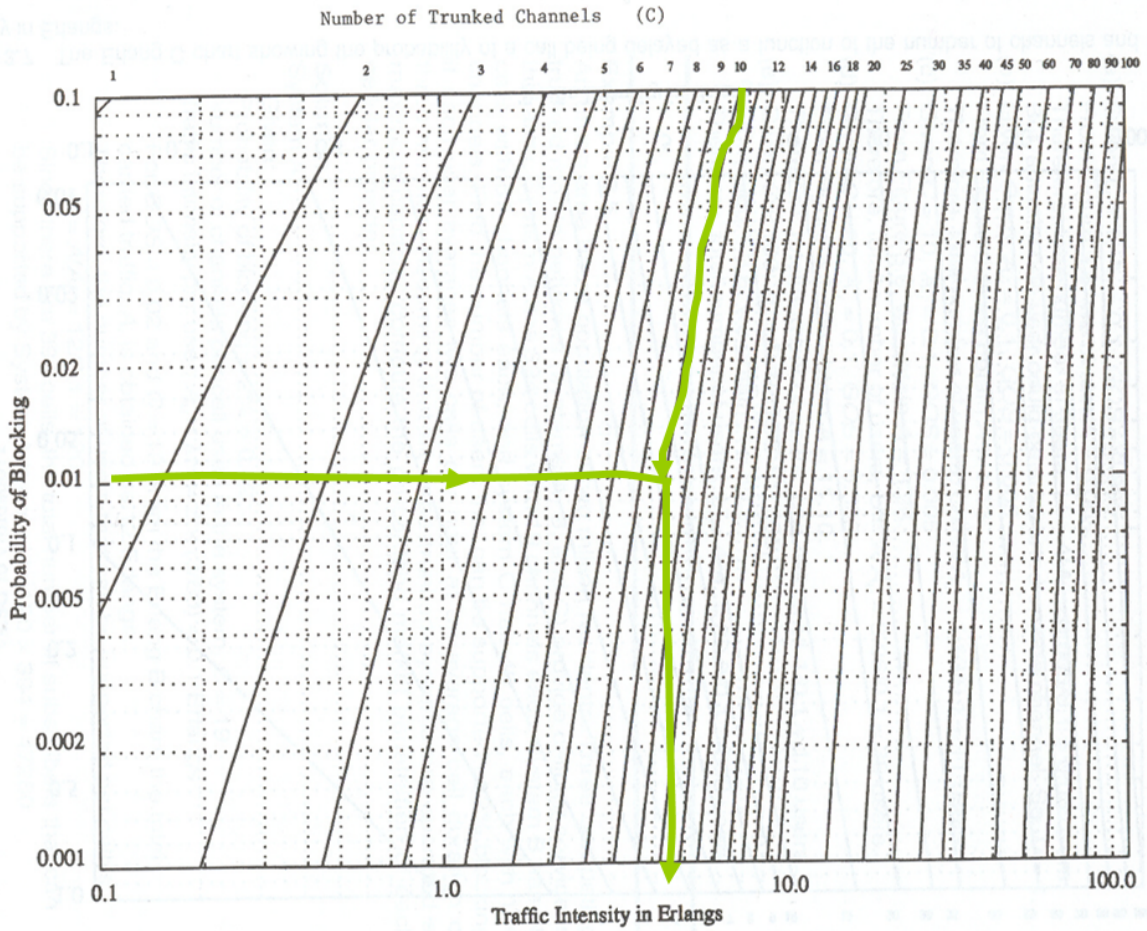
Erlang B has been widely used in evaluating blocking prob of telephone systems.

Trunking and Grade of Service (GoS)



Erlang B chart: showing prob of blocking as a fun of the # of channels & traffic intensity in Erlang

Trunking and Grade of Service (GoS)



Example: 2 For $C = 10$, $GoS = 0.01$, Then from the chart, $A = 4.46$

Trunking and Grade of Service (GoS)

Example 3: Capacity of an Erlang B system (Table 3.4 in text)

Number of Channels C	Capacity (Erlangs) for QoS=			
	0.01	0.005	0.002	0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

Example 4: The American AMPS cellular system is designed for GoS of 2% blocking. $\Longrightarrow$ 2 out of 100 calls are blocked during busiest hour.

Trunking and Grade of Service (GoS)

Example 5: A cellular system is designed for GoS of 0.5% blocking. How many users can be supported by the system, if blocked calls are cleared, for the following numbers of channels: (a) 5, (b) 10, (c) 20, (d) 100. Assume that each user generates 0.1 Erlangs of traffic.

Example 6: An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 20 channels each, system B has 98 cells with 70 channels each, and system C has 49 cells, each with 100 channels. (a) Find the number of users that can be supported at 1% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. (b) Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Hint: $A_a = 12, A_b = 45, A_c = 88$

Trunking and Grade of Service (GoS)

Example 7: A certain city has an area of 1,300 square miles and is covered by a US AMPS cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 42MHz of spectrum with a full duplex channel bandwidth of 60kHz. Assume a GoS of 1% for a blocked cleared system is specified. If the offered traffic per user is 0.03 Erlangs, compute:

1. the numbers of cells in the service area,
2. the numbers of channels per cell,
3. traffic intensity of each cell,
4. the maximum carried traffic,
5. the total number of users that can be served for 1% blocking probability,
6. number of mobiles per unique channel (taking into account the fact that channels are reused).

Trunking and Grade of Service (GoS)

(2) Blocked calls delayed models (Erlang C):

- Instead of clearing a call; put it in a queue (a first-in, first-out line).
- Calls will be processed in the order received.
- There are now two things to determine
  1. Prob a call will be delayed (enter queue) "stick around" & wait for 1st open Ch.
  2. The probability that the delay will be longer than t sec.
- Here GoS means the prob that a call will be forced into the queue AND it will wait longer than t seconds before being served (for some given t).
- We need a couple additional assumptions:
  1. The queue is infinitely long. This translates to infinite memory
  2. No one who is queued gives up / hangs up (rather than wait)
- The probability that a call will be delayed (Erlang C formula) is given by:

$$P(delay > 0) = \frac{A^C}{A^C + C!(1 - \frac{A}{C}) \sum_{k=0}^{C-1} \frac{A^k}{k!}}$$

Trunking and Grade of Service (GoS)

(2) Blocked calls delayed models (Erlang C):

- It is typically easiest to find a result from Figure 2 Slide 21.
- Once it enters the queue, the probability that the delay is greater than t (for $t > 0$) is given as

$$P(delay > t | delay > 0) = e^{-\frac{C-A}{H}t}$$

- The two combined are needed to find the marginal (overall) probability that a call will be delayed AND experience a delay greater than t, the event that we are quantifying in GoS.

$$GoS = P(lost) = P(delay > t) = P(delay > t | delay > 0) \times P(delay > 0)$$

$$GoS = P(delay > 0)e^{-\frac{C-A}{H}t}$$

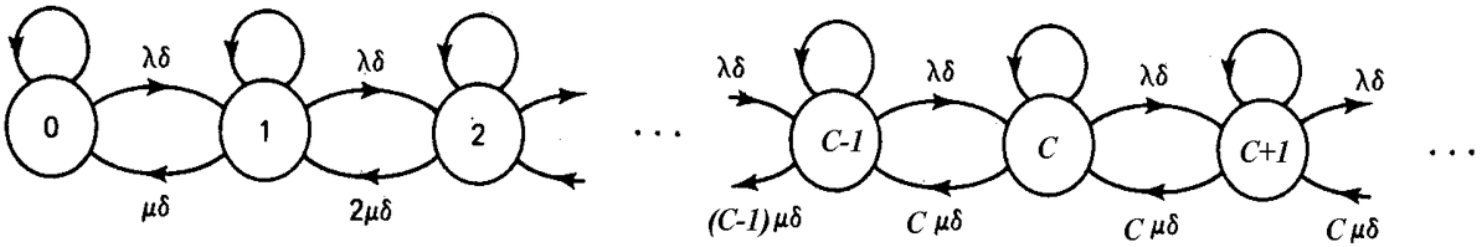
(2) Blocked calls delayed models (Erlang C): Proof [see appendix A in textbook]

Consider a cell with C trunked Chs and assume that if channels are unavailable in the cell to serve a call request, the call is not blocked immediately but rather placed on a queue until channel becomes available or maximum queuing time elapses.

Trunking and Grade of Service (GoS)

(2) Blocked calls delayed models (Erlang C): Proof [see appendix A in textbook]

The state transition diagram for the system is:



- Note that here state C+1 denotes the condition that all C channels are in use and a new call request arrives which is placed on queue.
- At steady state, we can write the global balance equation for the system.
- For case $n \leq C$, we have:

$$P_1 = P_0 \frac{\lambda}{\mu}, P_2 = P_1 \frac{\lambda}{2\mu}, ..., P_C = P_{C-1} \frac{\lambda}{C\mu}$$

Trunking and Grade of Service (GoS)

(2) Blocked calls delayed models (Erlang C): Proof [see appendix A in textbook]

- In General: $P_n = P_0(\frac{\lambda}{\mu})^n \frac{1}{n!}, \forall n \leq C$
- For the case $n > C$, we have:
  $P_{C+1} = P_C(\frac{\lambda}{C\mu}), P_{C+2} = P_{C+1}(\frac{\lambda}{C\mu}) = P_C(\frac{\lambda}{C\mu})^2, ...P_n = P_C(\frac{\lambda}{\mu})^{n-C}(\frac{1}{C})^{n-C}$
- So that in general for this case, we have: $P_n = P_0(\frac{\lambda}{\mu})^n \frac{1}{C!}(\frac{1}{C})^{n-C} \forall n > C$
- Also, $\sum_{n=0}^{C} P_n = 1$
- Thus
  $P_0 + P_0(\frac{\lambda}{\mu})\frac{1}{1!} + P_0(\frac{\lambda}{\mu})^2\frac{1}{2!} + ... + P_0(\frac{\lambda}{\mu})^C\frac{1}{C!} + P_0(\frac{\lambda}{\mu})^{C+1}\frac{1}{C!}(\frac{1}{C})^{(C+1-C)} + ... = 1$
- Therefore, $P_0 = [\sum_{n=0}^{C-1}(\frac{\lambda}{\mu})^n\frac{1}{n!} + \frac{1}{C!}(\frac{\lambda}{\mu})^C\sum_{n=C}^{\infty}(\frac{\lambda}{\mu})^{n-C}(\frac{1}{C})^{n-C}]^{-1}$
- The prob that an arriving call meets all C channels busy (and thus has to be placed on the Queue, i.e. delayed) is: $P(Queueing) = \sum_{m=C}^{\infty} P_n$
- Substituting for P_n & P_0 from above and since $A = \lambda H = \frac{\lambda}{\mu}$ and $\frac{\lambda}{C\mu} < 1$.

Trunking and Grade of Service (GoS)

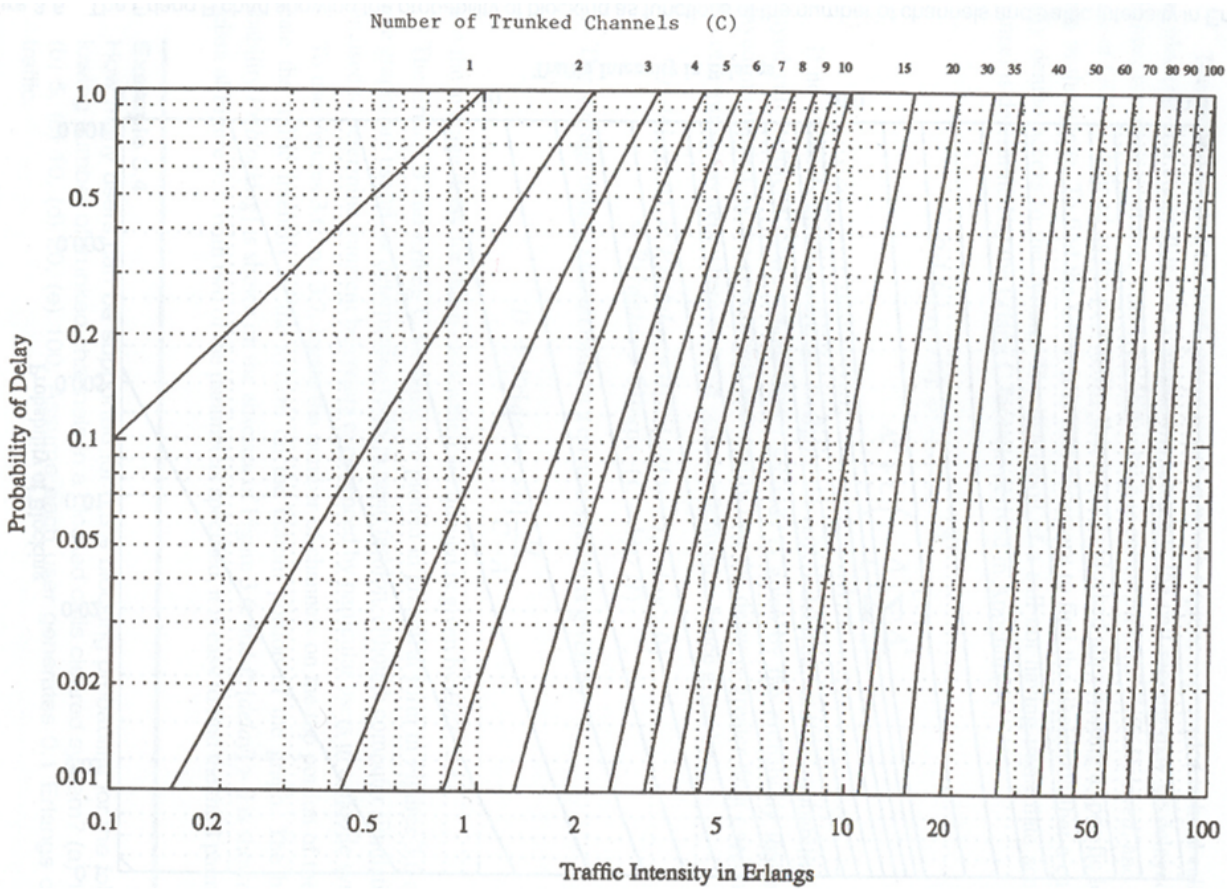
(2) Blocked calls delayed models (Erlang C): Proof [see appendix A in textbook]

- The final result will be: $P(Queueing) = \frac{\frac{1}{C!}(\frac{\lambda}{\mu})^C}{\frac{1}{C!}(\frac{\lambda}{\mu})^C + (1-\frac{\lambda}{C\mu})\sum_{k=0}^{C-1}(\frac{\lambda}{\mu})^k \frac{1}{k!}}$

- Then $P(Queueing) = \frac{A^C}{A^C + C!(1-\frac{A}{C})\sum_{k=0}^{C-1}\frac{A^k}{k!}} \Longrightarrow P(delay > 0)$ or Erlang C.
- GoS for Blocked calls delayed system is defined as the prob that a call is blocked after waiting for maximum queuing time in the system.
- The prob of a call lost:
  $P(delay > t) = P(delay > t | delay > 0) \times P(delay > 0)$
- Average # of calls on queue at any given time is given by: $N_Q = \frac{P_c \frac{\lambda}{C\mu}}{1-\frac{\lambda}{C\mu}}$
- P_C is the Prob. that a call is placed on queue.

Hint: $N_Q = 1 \times P_{C+1} + 2 \times P_{C+2} + \ldots$

Trunking and Grade of Service (GoS)



Erlang C chart: showing prob of a call being delayed as a fun of the # of channels & traffic intensity in Erlangs

Trunking and Grade of Service (GoS)

Example 8: A 7 cell cluster (i.e. N = 7) has 30 MHz allocated to it for forward channels and each channel is 200 kHz. Assume blocked-called-delayed and a probability of delay of 1%, and each user makes one 10 minute call every 3 hours.
(a) What is the number of users that can be supported?
(b) What is $P(delay > 10 seconds)$?
(c) What if it was a blocked-calls-cleared system with QoS of 1%?

Solution:

(a) 30 MHz / 200 kHz = 150 channels, divided among 7 cells, so about 20 channels per cell (after 1 control channel per cell).
With 20 channels, and probability of delay of 1%, looking at figure of Erlang C, we see A is about 11 Erlang. With $A = A_u U$ and $A_u = \lambda H = (\frac{1}{3*60}) \times 10 = 1/18$ Erlang, we have that U = 198 $\approx$ 200. But this is per cell. So there can be 7(200) = 1400 users in the 7 cells.
(b) in each cell, $C = 20, A = 11, H = 10 \times 60 = 600$ sec. So $P(delay > 10$ sec$)$ $= (0.01)e^{-\frac{(20-11)}{600}10} = 0.01e^{-0.15} = 0.0086$.
(c) From Figure of Erlang B, $A \approx 12$, so $U = 216$, for a total of 1512 total users.

Trunking and Grade of Service (GoS)

Example 9: A 4-cell reuse cellular system has a total of 60 unique channels used in the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) how many users will this system support?,

(b) what is the prob that a delayed call will wait for more than 10 sec on queue ?

(c) what is the prob that a call will be lost or cleared in this system ?

(Assuming maximum queuing time =10sec).

Improving Coverage and Capacity in Cellular Systems

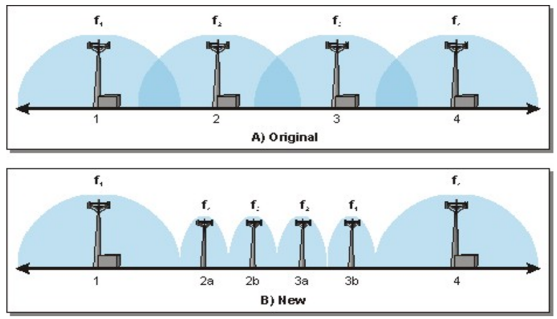
The # of channels in a cellular system may eventually become insufficient to service available subscribers as demand for services increase. Practical techniques that can be used to expand the capacity of cellular systems in such situations include the followings:

1. Cell splitting
2. Use of overlay structure (micro and umbrella cells):
3. Cell sectoring:
4. Advanced technologies for improving coverage
5. Diversity
6. Multiple-input multiple-output (MIMO)
7. Relays (or repeater) stations

Improving Coverage and Capacity in Cellular Systems

(1) Cell splitting

Is the process of sub-dividing a cell into smaller cells, each with its own BS and a corresponding reduction in antenna height and TX power (to achieve reduced coverage area per cell). When cells are split, the # of times that channels are reused throughout the service area increase, and this increases the capacity (# of users).

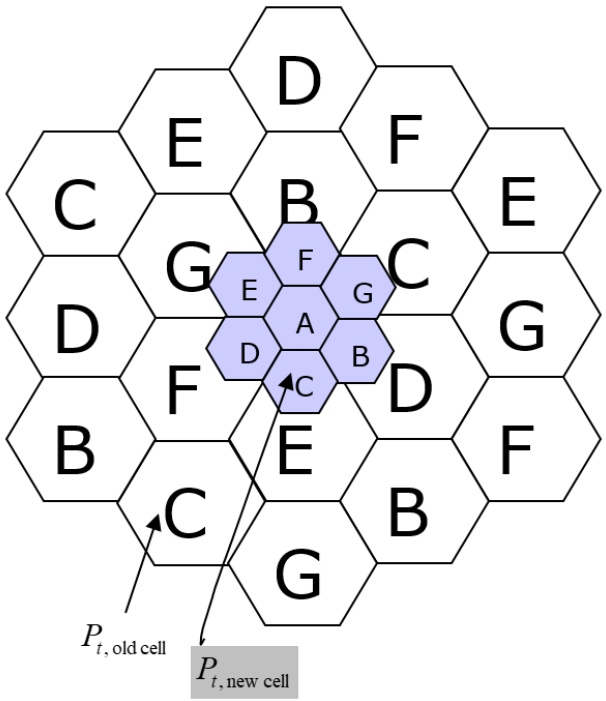


Various options can be used in practice to split a cell.

1. Antenna Downtilting: BS antennas are focused towards the ground rather than towards the horizon, to limit their radio coverage.
2. Reducing BS transmitted power: reducing the effective radiated power of the BS by a desired factor to achieve a desired coverage reduction.
3. Reducing Antenna height: cutting back the physical length of the mast.
4. Or any combinations of these

Improving Coverage and Capacity in Cellular Systems

Example 10: If cell splitting by power reduction is employed such that the radius of every cells are cut in half, with the smaller cells, approximately how many times as many cells would be required to cover the same service area?, and determine the BS transmit power for the new cells.

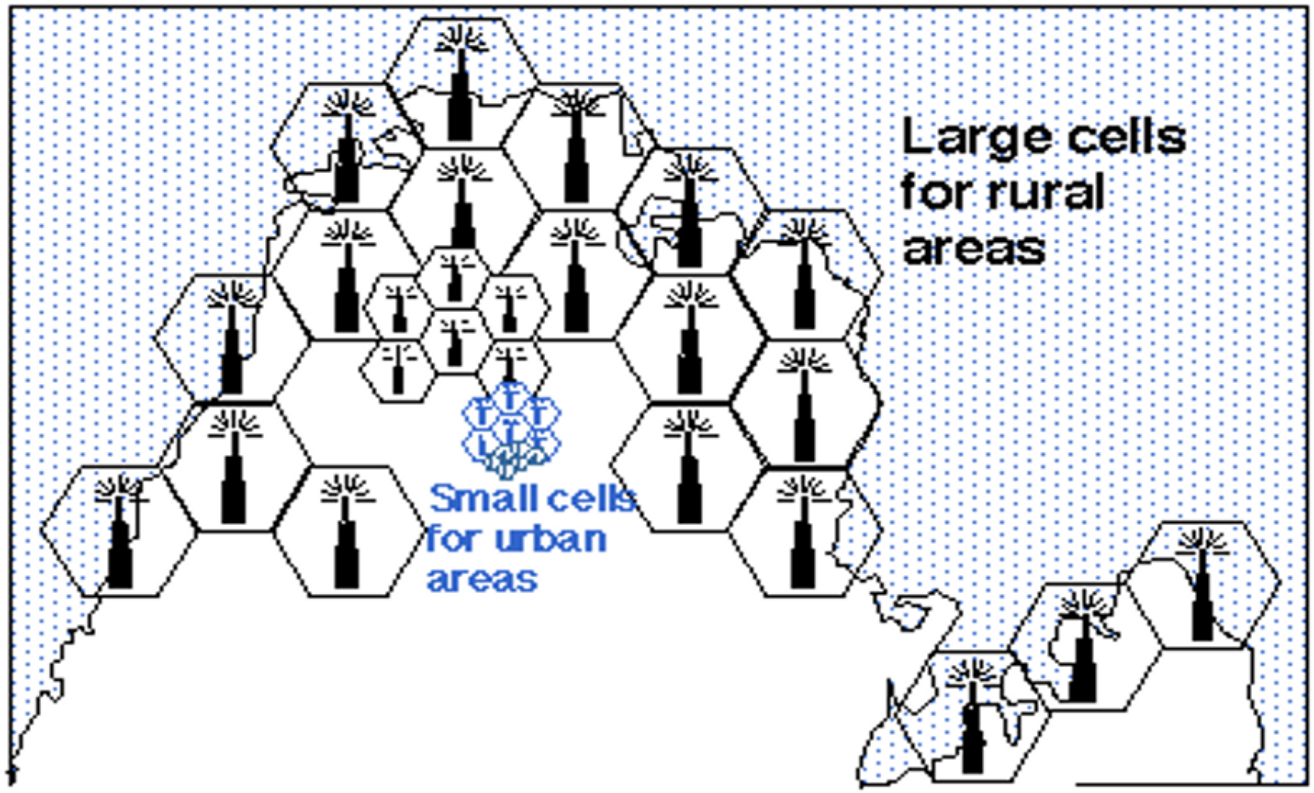


Solution: To keep SIR unchanged, we make sure that the received power at the edge of the new cell is same as the received power at the edge of the old cell. Thus, P_r (at new cell boundary) $\propto P_{t,new}R_{new}^{-n} = P_r$ (at old cell boundary) $\propto P_{t,old}R_{old}^{-n}$.

$$P_{t,new} = P_{t,old}(\frac{1}{2})^n$$

where $R_{new} = \frac{R_{old}}{2}$ and n: is the path loss exponent.

Improving Coverage and Capacity in Cellular Systems



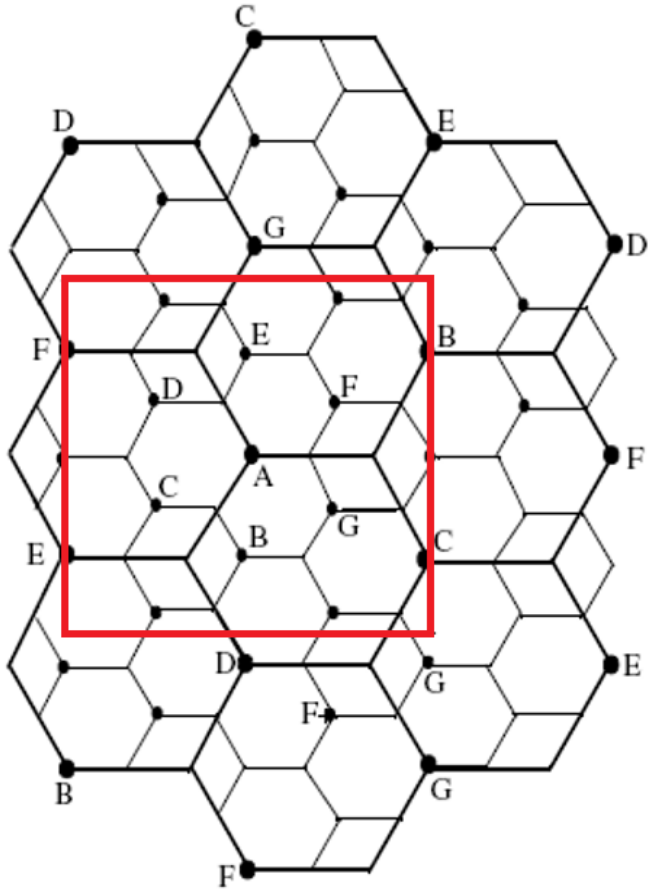
Mixed Cell sizes are employed in typical cellular systems, for different areas.

Large cells for rural areas, small cells for urban areas. Why?

Improving Coverage and Capacity in Cellular Systems

Example 11: Let each BS in the figure use 60 channels, regardless of cell size, and let $R_{macro-cell} = 1$ km & $R_{micro-cell} = 0.5$ Km be the radius of the macro- and micro-cells. Find the number of channels N contained in a 3 km by 3 km square centered around BS "A" under the following conditions:

1. Without the use of microcells
2. When the lettered microcells shown in the figure are used
3. If all the original BSs are replaced by microcells



Improving Coverage and Capacity in Cellular Systems

Solution: (1)

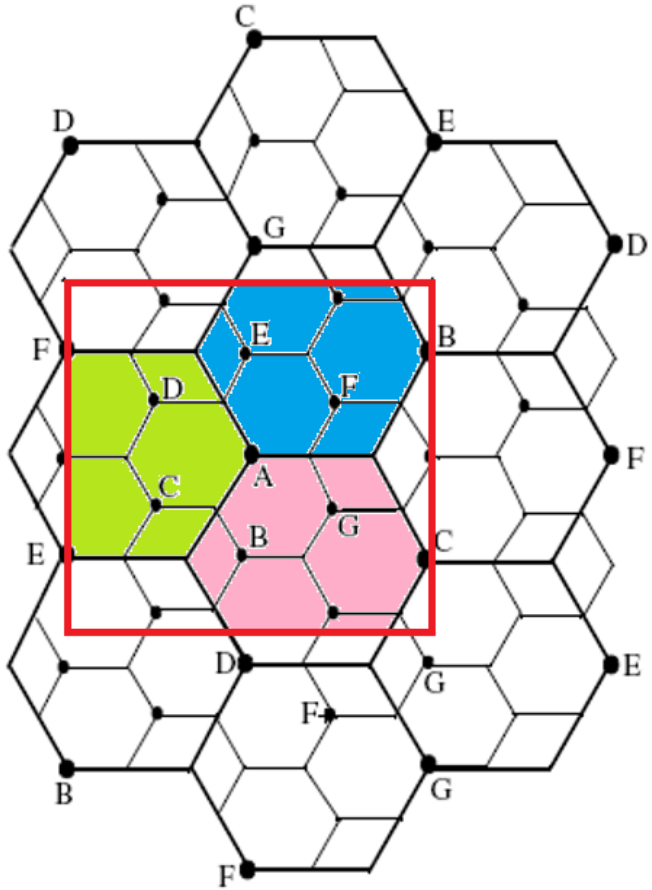
No micro-cells: $R = 1$ Km = sides of larger hexagon. Hence, we need to cover 1.5Km to the right, left, top, & bottom of BS "A".

(1) This means 5 BS covered approx.

(2) Hence $N = 5 \times 60 = 300$ channels

Solution: (2)

when the lettered microcells shown in the figure are used: BS "A" surrounded by 6 microcells. Hence, $N = (5 + 6) \times 60 = 660$ channels.

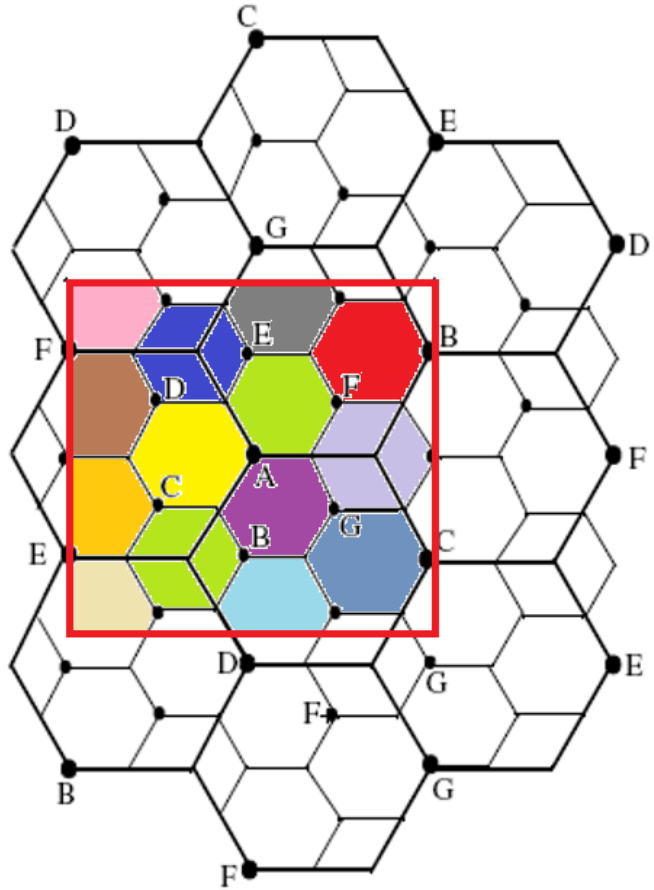


Incomplete cell is counted as one if its coverage area is roughly up to a BS sector.

Improving Coverage and Capacity in Cellular Systems

Solution: (3)
if all the original BSs are replaced by microcells:
approx. 18 microcells are covered in this case. Hence,
$N = (18) \times 60 = 1080$ channels.

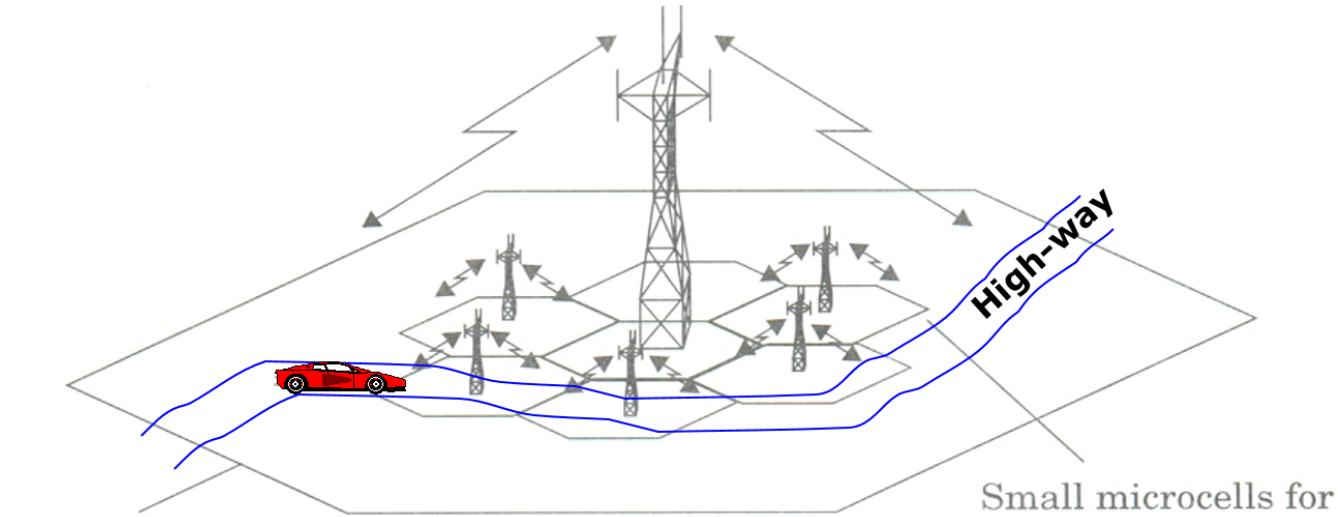
Incomplete cell is counted as one if its coverage area is roughly up to a BS sector.



Improving Coverage and Capacity in Cellular Systems

(2) Use of overlay structure (micro and umbrella cells):

An overlay structure combines cells of different sizes. An umbrella (large) cell provides coverage for large area (services high speed mobiles). Within the coverage area are micro cell base stations servicing other mobiles. This arrangement helps to reduce numbers of handoffs and allow efficient use of radio resources, which can be exploited to increase capacity.

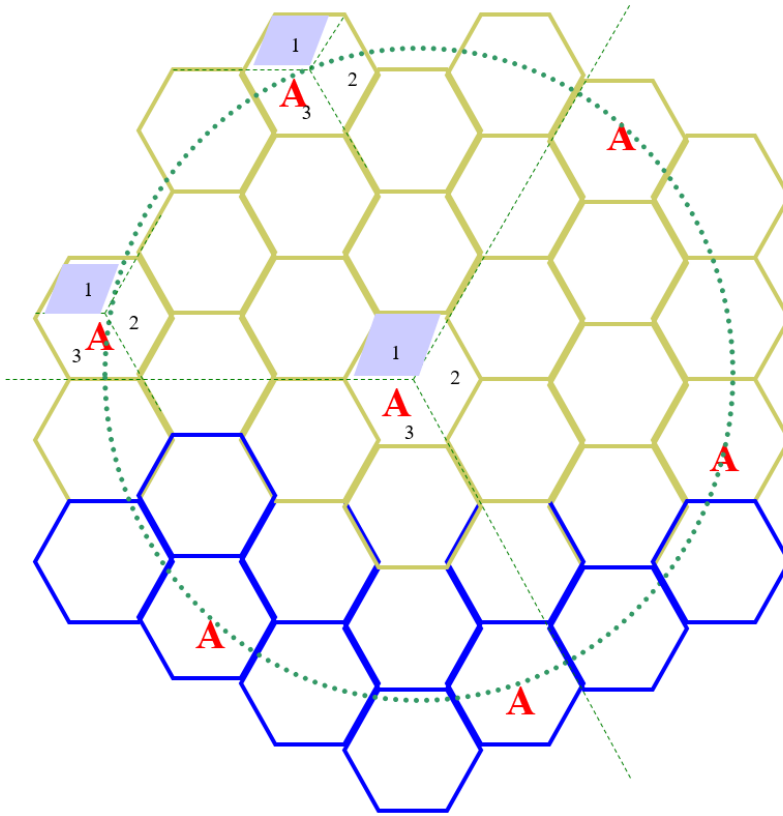


Improving Coverage and Capacity in Cellular Systems

(3) Cell sectoring:

- Single omni-directional antenna at BS is replaced by several directional antennas, each covering a sector of the original cell.
- The SIR is improved since directional antenna cause interference to less # of neighboring (co-channels) cells.
- Capacity improvement is achieved by reducing cluster size (i.e. increasing freq reuse factor).
- The factor by which co-channel interference is reduced depends on the amount of sectoring used.
- 120^o sectoring (3 sectors/cell) or 60^o sectoring (6 sectors/cell) are normally used.
- For 7-cell reuse pattern, the # of interferers in the 1^{st} tier is reduced from 6 to 2 if 120^o sectoring is used. This leads to reduction in CCI and consequently the SIR improves.

Channels used in a particular cell are broken down into sectored groups and are used only within a particular sector.



Improving Coverage and Capacity in Cellular Systems

Example 12:

For a 7-cell reuse cellular system, assuming that the distances between all the interfering co-channel BSs and the mobile is D and they are equal. Assume $n = 3$, estimate the SIR for:
(a) Case when omni directional antenna is used,
(b) 120^0 sectoring is used.

What is the effect of sectoring on trunking efficiency?

Improving Coverage and Capacity in Cellular Systems

(4) Advanced Technologies for improving coverage

See Ch 5 for details.

(5) Diversity techniques

Increases the quality of the received signal, which can be exploited to increase capacity e.g. by reducing the reuse distance.

All 2.5G/3G cellular systems have deployed 2-branch Diversity at base stations.

(6) Multiple-input multiple-output (MIMO) system

Increases the coverage/capacity of wireless links by employing coding techniques appropriate to multiple antenna transmission (and reception).

(7) The use of Relays (or repeater) stations

It provide range extension to places hard-to-reach (within buildings). It are bidirectional (i.e., work on both forward and reverse links) to amplify and re-radiate signals to and from serving BSs. It Does not add capacity, but just re-radiates signals to specific locations.

HomeWork 2

Q1

Derive the Eq. given for the worst-case SIR in cellular system with $N = 7$ above, which was obtained considering only the first layer of interfering cells, and assuming that adjacent BS is of distance D from the mobile.

Q2

Now consider 2^{nd} Tier of interfering cells and derive new, more accurate expression for the SIR for same system. [Application]: WiMAX cellular deployment in unlicensed bands must consider all significant tiers of interferers to match theoretical analysis with real-time measurement for SIR. Analysis using 1^{st} Tier only won't be sufficient because there are already un-known interferers, thus all known ones must be accurately predicted and planned for.

Term Project

Toptic Proposal due: Week # 6

Final Report/Presentations: Week # 12

Each Group: 5 Students

All Groups = 12 Group

Project Topics:

Group ID	Topic
1	Survey of 4G-LTE networks
2	Survey of 5G cellular networks
3	Millimeter wave bands for 5G cellular systems
4	Multi-input multi-output (MIMO) systems
5	Space time block codes (STBC)
6	Polar codes
7	Diversity techniques
8	Code division multiple access (CDMA)
9	Orthogonal frequency division multiplexing (OFDM)
10	Filter bank multi-carriers (FBMC)
11	Non-orthogonal multiple access (NOMA)
12	Sparse code multiple access (SCMA)